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On a functional equation related to distributivity of fuzzy implications

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On a functional equation related to distributivity of fuzzy implications

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Under the supervision of:
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1 Introduction

Recently in some considerations connected to the distributivity laws of fuzzy implications over triangular norms and conorms, the following equation appeared

$$f(\min(x+y,a)) = \min(f(x)+f(y),b).$$

In the current paper we discuss results, which dr hab. Michał Baczyński, dr Tomasz Szostok and me have obtained recently during our studies on a generalized version of this equation, namely the equation

$$f(m_1(x+y)) = m_2(f(x)+f(y)), \tag{1}$$

where m_1, m_2 are functions defined on some intervals of $\mathbb{R}$ satisfying additional assumptions.

However, this paper was intended also to present some background of our studies. Therefore In this section we introduce contents of the paper, few notations and some motivations for our studies. The next Section is devoted to the derivation of the particular functional equation we have examined with dr hab. Michał Baczyński and dr Tomasz Szostok.

The solutions to the equation (1), which we've obtained, are presented in Sections 3 and 4 - in the case when function m_2 is injective and in the case when function m_2 isn't injective.

Section 5 contains two examples of solutions to the equation (1) for specific functions m_1 and m_2. Finally in the last Section we present a brief summary of that paper. Some open problems are mentioned and some applications are indicated.

Those are four tautologies in classical logic involving the implication:

$$(x \vee y) \to z \equiv (x \to z) \wedge (y \to z) \qquad (T1)$$
$$(x \wedge y) \to z \equiv (x \to z) \vee (y \to z) \qquad (T2)$$
$$x \to (y \wedge z) \equiv (x \to y) \wedge (x \to z) \qquad (T3)$$
$$x \to (y \vee z) \equiv (x \to y) \vee (x \to z) \qquad (T3)$$

They are useful when we want to simplify rule bases, for instance we may have such rule base

$$x_1 \wedge y_1 \to z_1,$$
$$x_1 \wedge y_2 \to z_2,$$
$$x_2 \wedge y_1 \to z_3,$$
$$x_2 \wedge y_2 \to z_4,$$

and we can simplify it according to the above tautologies - we begin with braking each rule according to (T2) and next we join pairs of rules with the same antecedent according to (T4):

$$\begin{array}{ccccccc} x_1 \wedge y_1 \to z_1, & \Rightarrow & (x_1 \to z_1) \vee (y_1 \to z_1), & \Rightarrow & x_1 \to (z_1 \vee z_2), & \Rightarrow & x_1 \to z_2, \\ x_1 \wedge y_2 \to z_2, & & (x_1 \to z_2) \vee (y_2 \to z_2), & & x_2 \to (z_3 \vee z_4), & & x_2 \to z_4, \\ x_2 \wedge y_1 \to z_3, & & (x_2 \to z_3) \vee (y_1 \to z_3), & & y_1 \to (z_1 \vee z_3), & & y_1 \to z_3, \\ x_2 \wedge y_2 \to z_4 & & (x_2 \to z_4) \vee (y_2 \to z_4) & & y_2 \to (z_2 \vee z_4) & & y_2 \to z_4 \end{array}$$

-

The last simplification is possible if we additionally know the relations between z_1, z_2, z_3 and z_4, for instance that we have such inclusions $z_1 \subseteq z_2 \subseteq z_3 \subseteq z_4$, which means such implications: $z_1 \to z_2, \quad z_2 \to z_3, \quad z_3 \to z_4$.

If we allow to attribute to $x_1, x_2, y_1, y_2, z_1, z_2$ other truth values than only 0 and 1, which is often convenient in real life, we need more general logic operators than classical conjunction, disjunction and implication. Then one of possibilities is to use fuzzy operators, which are defined on the whole unit square and behave the same as classical ones on the endpoints of the square. We will remind briefly the definitions of fuzzy implications (generalization of classical implication), t-norms (generalization of classical conjunction) and t-conorms (generalization of classical disjunction).

Definition. A function $I : [0,1]^2 \to [0,1]$ is called ***a fuzzy implication*** if it satisfies, for all $x, y \in [0,1]$, the following conditions

1. $I(\cdot, y)$ is decreasing,
2. $I(x, \cdot)$ is increasing,
3. $I(0,0) = 1$,
4. $I(1,1) = 1$,
5. $I(1,0) = 0$.

Definition. A function $T : [0,1]^2 \to [0,1]$ is called ***a triangular norm (t-norm)*** if it satisfies, for all $x, y, z \in [0,1]$, the following conditions

1. $T(x,y) = T(y,x)$,
2. $T(x, T(y,z)) = T(T(x,y), z)$,
3. $T(x, \cdot)$ is decreasing,
4. $T(x,1) = x$.

Definition. A function $S : [0,1]^2 \to [0,1]$ is called ***a triangular conorm (t-conorm)*** if it satisfies, for all $x, y, z \in [0,1]$, the following conditions

1. $S(x,y) = S(y,x)$,
2. $S(x, S(y,z)) = S(S(x,y), z)$,
3. $S(x, \cdot)$ is increasing,
4. $S(x,0) = x$.

If we now rewrite tautologies (T1)-(T4) using fuzzy operators, we get equations:

$$I(S(x,y),z) = T(I(x,z), I(y,z)) \qquad (D1)$$
$$I(T(x,y),z) = S(I(x,z), I(y,z)) \qquad (D2)$$
$$I(x,T(y,z)) = T(I(x,y), I(x,z)) \qquad (D3)$$
$$I(x,S(y,z)) = S(I(x,y), I(x,z)) \qquad (D4)$$

However, in the contradiction to the classical tautologies, those equations aren't satisfied for any fuzzy operators. That is why we need to study them.

The importance of such equations in Fuzzy Control and Fuzzy Systems has been first emphasized by Combs and Andrews [3], wherein they exploit equation (D2) in their inference mechanism towards reduction in the complexity of fuzzy ,,IF-THEN" rules. Subsequently, there were many discussions, most of them pointing out the need for a theoretical investigation required for employing such equations.

In our studies we focused on equation (D4) with two t-conorms S_1, S_2 instead of one t-conorm S

$$I(x, S_1(y,z)) = S_2(I(x,y), I(x,z)) \qquad (D4')$$

We have limited our studies only to Archimedean and continuos t-conorms. The next Section contains derivation of equation (1) from equation (D4') in this case.

2 Derivation of equation (1)

Definition. A t-conorm S is called ***Archimedean***, if for all $x, y \in (0,1)$ there exists an $n \in \mathbb{N}$ such that

$$\underbrace{S(x, S(x, \dots S(x,x)))}_{n \text{ times } x} > y$$

(which we denote also by $x_S^{[n]} > y$).

The form of every continuous and Archimedean t-conorm is well known. The following Lemma points this form.

Lemma 1. *When S is continuous and Archimedean triangular conorm, then S is of the form*

$$S(x,y) = s^{-1}(\min(s(x) + s(y), s(1))), \quad x, y \in [0,1]$$

where $s : [0,1] \to [0,\infty]$ is continuous, strictly increasing function with $s(0) = 0$.

Using the representation from above Lemma the equation (D2) takes form

$$I(x, s_1^{-1}(\min(s_1(y) + s_1(z), s_1(1)))) = s_2^{-1}(\min(s_2(I(x,y)) + s_2(I(x,z)), s_2(1)))$$
$$\Leftrightarrow \quad s_2(I(x, s_1^{-1}(\min(s_1(y) + s_1(z), s_1(1))))) = \min(s_2(I(x,y)) + s_2(I(x,z)), s_2(1))$$
$$\Leftrightarrow \quad g_x(\min(s_1(y) + s_1(z), s_1(1))) = \min(g_x(s_1(y)) + g_x(s_1(z)), s_2(1)),$$

where $g_x(\,\cdot\,) = s_2 \circ I(x, s_1^{-1}(\,\cdot\,))$, for a fixed $x \in [0,1]$.

The last equation we can write down in such form

$$g_x(\min(u+v, s_1(1))) = \min(g_x(u) + g_x(v), s_2(1)),$$

where $u, v \in [0, s_1(1)]$, and g_x is an unknown function.

Thus, in the paper [2], authors have found the general form of $f\colon [0, r_1] \to [0, r_2]$ (for fixed $r_1, r_2 \in (0, \infty)$) satisfying the functional equation

$$f(\min(x+y, r_1)) = \min(f(x) + f(y), r_2).$$

In this paper we consider the generalized version of this equation i.e., we replace functions $\min(\cdot, r_1)$, $\min(\cdot, r_2)$ occurring directly in this equation, by functions m_1, m_2 satisfying some assumptions. This means that we will study the equation

$$f(m_1(x+y)) = m_2(f(x) + f(y)). \tag{2}$$

Moreover, we shall not only find the general form of a function f, but we shall also prove that functions m_1 and m_2 must satisfy some special properties, if we want the equation (2) to have some nontrivial solutions f.

3 Solutions of equation (1) when m_2 is injective

First we consider the situation when m_2 is injective (in particular it is a bijection).

Lemma 2. *Let $r_1, r_2 \in (0, \infty)$ be some numbers and let $m_1\colon [0, 2r_1] \to [0, r_1]$, $m_2\colon [0, 2r_2] \to [0, r_2]$ be given functions. If m_2 is injective and a function $f\colon [0, r_1] \to [0, r_2]$ satisfies the functional equation* (1)*, then*

$$2f\left(\frac{x+y}{2}\right) = f(x) + f(y), \qquad x, y \in [0, r_1]. \tag{3}$$

Proof. From (1) we obtain

$$m_2^{-1}\left(f(m_1(x+y))\right) = f(x) + f(y), \qquad x, y \in [0, r_1],$$

and putting $F(t) := m_2^{-1}(f(m_1(t)))$, for $t \in [0, 2r_1]$ we get

$$F(x+y) = f(x) + f(y), \qquad x, y \in [0, r_1]. \tag{4}$$

Now, if we take any $x, y \in [0, r_1]$, then from (1) we have

$$f(x) + f(y) = F(x+y) = F\left(\left(\frac{x+y}{2}\right) + \left(\frac{x+y}{2}\right)\right) = 2f\left(\frac{x+y}{2}\right),$$

and therefore f satisfies (3). ∎

Theorem 1. *Let $r_1, r_2 \in (0, \infty)$ be some numbers and let $m_1\colon [0, 2r_1] \to [0, r_1]$, $m_2\colon [0, 2r_2] \to [0, r_2]$, $f\colon [0, r_1] \to [0, r_2]$ be given functions. Further, let m_2 be injective. Then the following sentences are equivalent:*

(i) The triple of functions m_1, m_2, f *satisfies the equation* (1).

(ii) Either $f = b$ *for some* $b \in [0, r_2]$ *and* $m_2(2b) = b$*, or* $f(x) = ax + b$ *for some* $a, b \in \mathbb{R}$, $a \neq 0$ *such that*

$$ax + b \in [0, r_2], \qquad \text{for all } x \in [0, r_1] \tag{5}$$

and

$$m_1(x) = \frac{m_2(ax + 2b) - b}{a}. \tag{6}$$

Proof. $(ii) \implies (i)$ Now it is easy to check that these functions satisfy (1). Indeed, in the case $f(x) = b$ our equation is satisfied provided that $m_2(2b) = b$. On the second case for every $x, y \in [0, r_1]$ we have

$$\begin{aligned} f(m_1(x+y)) = am_1(x+y) + b = a\frac{m_2(a(x+y) + 2b) - b}{a} + b \\ = m_2(ax + b + ay + b) = m_2(f(x) + f(y)). \end{aligned}$$

$(i) \implies (ii)$ From Lemma 2 we obtain that f satisfies the Jensen equation (3). However, since f is bounded, there exist $a, b \in \mathbb{R}$ such that $f(x) = ax+b$ (see [5], Theorem III.2.2). If we consider the case $a = 0$, then $f(x) = b$ for all $x \in [0, r_1]$ and from (1) we obtain that $m_2(2b) = b$. If we assume that $a \neq 0$, then using the form of f in (1) we have

$$am_1(x + y) + b = m_2(ax + b + ay + b)$$

and taking here $y = 0$ we obtain

$$am_1(x) + b = m_2(ax + 2b)$$

which yields the equality (6). Clearly, the condition (5) must be satisfied, since f is defined on $[0, r_1]$ and takes values in $[0, r_2]$. □

4 Some solutions of equation (1) when m_2 is not injective

The case when function m_2 isn't injective, is more complicated and till now we know only partial characterization of solutions of equation (1). We discuss our theorem, which we are working on, in this Section, without the proof. We put some limitation to the form of function m_2, thus we haven't examined equation (1) for all not injective functions m_2 yet.

Theorem 2. *Let* $r_1, r_2 \in (0, \infty)$ *be some numbers and let functions* $m_1 \colon [0, 2r_1] \to [0, r_1]$, $m_2 \colon [0, 2r_2] \to [0, r_2]$ *be continuous and strictly increasing on some intervals* $[0, x_1]$, $[0, x_2]$*, respectively, and then be equal to* r_1, r_2*, respectively, where* $x_1 \leq r_1$ *and* $x_2 \leq r_2$*. Further, let* m_1, m_2 *satisfy*

$$m_1(0) = 0, \qquad 2m_1(x) > x, \qquad x \in (0, 2r_1) \tag{7}$$

and

$$m_2(0) = 0, \qquad 2m_2(x) > x, \qquad x \in (0, 2r_2). \tag{8}$$

Finally let f *be a function* $f \colon [0, r_1] \to [0, r_2]$.
If those assumptions are satisfied and triple of functions m_1, m_2, f *satisfies the equation* (1)*, then we have one of the following possibilities:*

(i) $f = r_2$ *and* m_1, m_2 *may be any functions;*

(ii) $f = 0$ *and* m_1, m_2 *may be any functions;*

(iii) $f(0) = 0, f(x) > x_2$ *for* $x > 0, f(r_1) = r_2,$ *and* m_1, m_2 *may be any functions;*

(iv) there exists $x_0 \in (0, r_1]$ *such that* $f(x) \geq x_2$ *for* $x \geq x_0$, $f(x) = r_2$ *for* $x \in [m_1(x_0), r_1]$ *and* $f(x) = \frac{x_2}{x_0}x$ *for* $x < x_0$. *Moreover in this case*

$$m_1(x) = \frac{x_0 m_2(\frac{x_2}{x_0}x)}{x_2}$$

for $x < y_0$, *such that* $m_1(y_0) = x_0$.

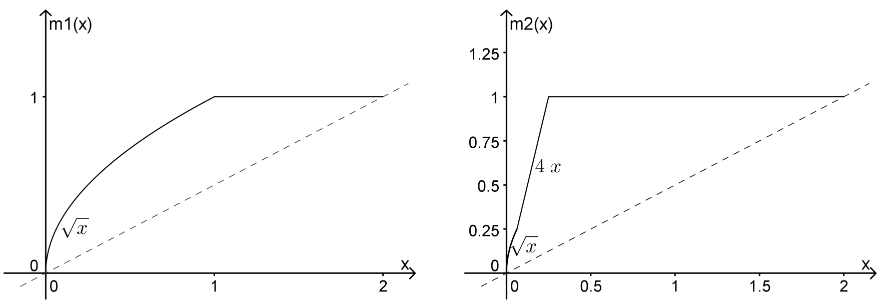
Conversely, if we add to (iv) an assumption that $y_0 = x_0$ *or* $f(m_1(x)) = m_2(f(x))$ *for* $x \in [y_0, x_0)$, *then each of the triples of functions described above satisfies the equation* (1)

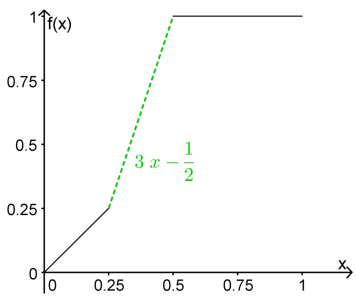
Remark. We will show that an additional assumption in the converse to Theorem 2 ($y_0 = x_0$ or $f(m_1(x)) = m_2(f(x))$ for $x \in [y_0, x_0)$) is necessary that is, we will point out a triple of functions m_1, m_2, f, such that have all the properties enumerated in (iv) of the last theorem, but the functional equation (1) does not hold.

Let $r_1 = r_2 = 1$ and $m_1(x) = \min(\sqrt{x}, 1)$ for $x \in [0, 2]$.

Next let $m_2(x) = \begin{cases} \sqrt{x}, & x \leq \frac{1}{16} \\ 4x, & \frac{1}{16} < x \leq \frac{1}{4} \\ 1, & \frac{1}{4} < x \leq 2 \end{cases}$ and $f(x) = \begin{cases} x, & x < \frac{1}{4} \\ 3x - \frac{1}{2}, & \frac{1}{4} < x \leq \frac{1}{2} \\ 1, & \frac{1}{2} < x \leq 1 \end{cases}$.

Proof. .





Thus $x_1 = 1$ and $x_2 = \frac{1}{4}$. One easily check that m_1 and m_2 satisfy assumptions of Theorem 2. It is easy to see that also $x_0 = \frac{1}{4}$. Now we check that the above functions satisfy the conditions given in (iv) in Theorem 2. Of course $f(x) \geq \frac{1}{4}$ for $x \geq \frac{1}{4}$. We see that $f(x) = 1$ for $x \in [\frac{1}{2}, 1]$ and since $m_1(\frac{1}{4}) = \frac{1}{2}$ we get $f(x) = r_2$ for $x \in [m_1(x_0), r_1]$. Also $f(x) = \frac{x_2}{x_0}x$ for $x < x_0 = \frac{1}{4}$. Finally, $m_1(\frac{1}{16}) = \frac{1}{4}$ and

$$\frac{x_0 m_2(\frac{x_2}{x_0}x)}{x_2} = \frac{\frac{1}{4}m_2(x)}{\frac{1}{4}} = m_2(x) = \sqrt{x} = m_1(x)$$

for all $x < \frac{1}{16}$.

However, for $x + y \in (y_0, x_0)$, that is for $x + y \in (\frac{1}{16}, \frac{1}{4})$ the equation (1)

$$f(m_1(x + y)) = m_2(f(x) + f(y))$$

does not hold. Indeed, for example for $x = y = \frac{1}{16}$ we have

$$f(m_1(\frac{1}{16} + \frac{1}{16})) = f(m_1(\frac{1}{8})) = \sqrt{\frac{1}{8}} = \frac{\sqrt{2}}{4}$$

while

$$m_2(f(\frac{1}{16}) + f(\frac{1}{16})) = m_2(\frac{1}{16} + \frac{1}{16}) = m_2(\frac{1}{8}) = \frac{1}{2}$$

□

We conclude that in order to obtain the equivalence in Theorem 2, we have to add an artificial condition to the case (iv) that $x_0 = y_0$ or simply that (1.1) is satisfied in the interval $[y_0, x_0)$. The question of a complete characterization of the solutions of (1.1) remains open.

5 Examples

Lemma 3. *In the case (iv) in Theorem 2, we know additionally that f must be continuous and increasing in its whole domain $[0, r_1]$*
(more precisely: for $x \in [0, m_1(x_0)]$ the function is strictly increasing and for $x \in [m_1(x_0), r_1]$ the function is constant).

Proof. For $x \in [0, x_0]$ function $f(x) = kx$ is continuous and strictly increasing. For $x \in [m_1(x_0), r_1]$ function $f(x) = r_2$ is constant.
Thus we only have to show that the function f is strictly increasing and continuous on the interval $[x_0, m_1(x_0)]$. Let $y_1, y_2 \in [x_0, m_1(x_0)], y_1 < y_2$. The function m_1 is continuous and strictly increasing on $[0, x_0]$, so there exist $z_1, z_2 \in [0, x_0]$, such that $m_1(z_1) = y_1, m_1(z_2) = y_2$ and $z_1 < z_2$.
In the case (iv) of Theorem 2 the following equation is satisfied

$$f(m_1(x)) = m_2(f(x)).$$

Thus $f(y_1) = f(m_1(z_1)) = m_2(f(z_1)) = m_2(kz_1)$ and $f(y_2) = f(m_1(z_2)) = m_2(f(z_2)) = m_2(kz_2)$.
Thus we have

$$f(y_1) < f(y_2) \Leftrightarrow m_2(kz_1) < m_2(kz_2) \Leftrightarrow kz_1 < kz_2 \Leftrightarrow z_1 < z_2,$$

what ends the proof of f being strictly increasing.

Similarly the continuity of f on the interval $[x_0, m_1(x_0)]$ results from the continuity of functions m_1, m_2 on their domains and f on the interval $[0, x_0]$, the fact that composition of continuous functions is continuous and from the equation $f(m_1(x)) = m_2(f(x))$.

□

Example 3. Let us fix arbitrarily $r_1, r_2 > 0$ and $\alpha \geq 1$. Let us consider the situation when $m_1(x) = \min(\alpha x, r_1)$ for $x \in [0, 2r_1]$ and $m_2 = \min(\alpha x, r_2)$ for $x \in [0, 2r_2]$. In this case we obtain the following equation

$$f(\min(\alpha(x + y), r_1)) = \min(\alpha(f(x) + f(y)), r_2).$$

Let us see that from Theorem 2 we obtain the following solutions: $f = r_2$ or $f = 0$, or $f(x) = \begin{cases} 0, & x = 0 \\ \geq x_2, & x > 0 \\ r_2, & x = r_1 \end{cases}$,
or $f(x) = \min(kx, r_2)$, where $k = \frac{r_2}{\alpha x_0}$.

We only need to show that in the case (iv) the only solution is $f(x) = \min(kx, r_2)$.
We have

$$x_i = \min\{x \in [0, r_i] : m_i(x) = r_i\} \text{ for } i = 1, 2 \quad \Rightarrow$$
$$x_1 = \frac{r_1}{\alpha}, x_2 = \frac{r_2}{\alpha} \text{ and } k = \frac{x_2}{x_0} = \frac{r_2}{\alpha x_0}$$

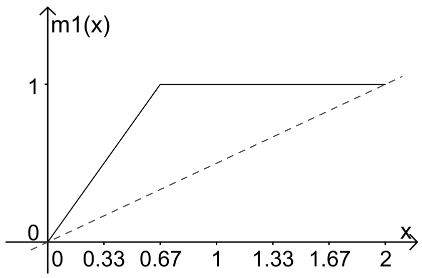
In this case from $f(m_1(x)) = m_2(f(x))$ we obtain the following equation

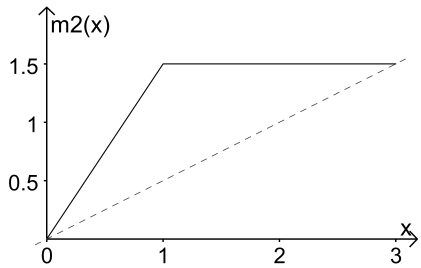
$$f(\min(\alpha x, r_1)) = \min(\alpha f(x), r_2)$$

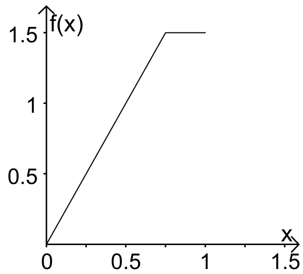
- Let $x < x_0$. Then $\min(\alpha f(x), r_2) = \min(\alpha kx, r_2) = \alpha kx$, because $\alpha kx = \alpha \frac{r_2}{\alpha x_0} x = \frac{x}{x_0} r_2 < r_2$
- Let $x < x_1$. Then $f(\min(\alpha x, r_1)) = f(\alpha x)$, because $\alpha x < \alpha x_1 = \alpha \frac{r_1}{\alpha} = r_1$.

Thus for $x < \min(x_0, x_1)$ we obtain $f(\alpha x) = \alpha k x$, which means that for $y < \min(\alpha x_0, \alpha x_1) = \min(\alpha x_0, r_1)$ we have $f(y) = ky$.
If $r_1 < \alpha x_0$ then $f(y) = ky$ for $y \in [0, r_1]$, thus $f(r_1) = kr_1 = \frac{r_1}{\alpha x_0} r_2 < r_2$, which is a contradiction.
Thus $\alpha x_0 \leq r_1$ and for $y < \alpha x_0$ we have $f(y) = ky$. We know from the Lemma 3, that function f is continuous and increasing, so $f(\alpha x_0) = k\alpha x_0 = \frac{r_2}{\alpha x_0} \alpha x_0 = r_2$ and $f(y) = r_2$ for $y > \alpha x_0$.
Finally we obtain $f(x) = \min(kx, r_2)$.

The plots of functions m_1, m_2 and f with $r_1 = 1, r_2 = \frac{3}{2}$ and $\alpha = \frac{3}{2}$ are presented below.







Example 4. Let us fix arbitrarily $r_1, r_2 > 0$ and let $m_1(x) = \min(\sqrt{r_1 x}, r_1), m_2(x) = \min(\sqrt{r_2 x}, r_2)$.
In this case we obtain the following equation

$$f(\min(\sqrt{r_1(x+y)}, r_1)) = \min(\sqrt{r_2(f(x) + f(y))}, r_2) \tag{9}$$

and from Theorem 2 we obtain that only nontrivial continuous solution is

$$f(x) = \frac{r_2}{r_1} x.$$

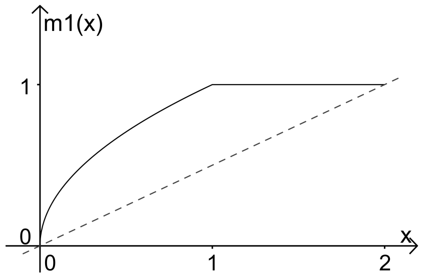
We obtain $x_1 = r_1$ and $x_2 = r_2$ from the form of m_1 and m_2.
That only one nontrivial solution appears in the case (iv) of Theorem 2. For $y = 0$ the equation (9) gives:

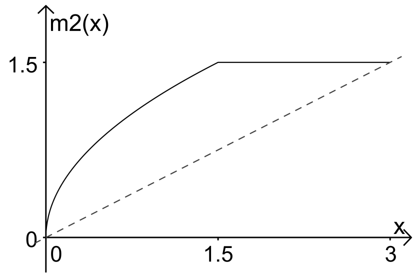
$$f(\min(\sqrt{r_1 x}, r_1)) = \min(\sqrt{r_2 f(x)}, r_2).$$

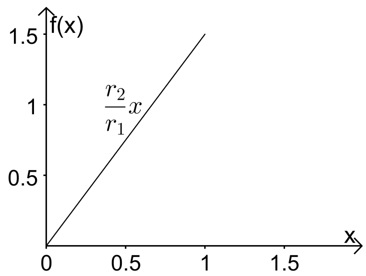
Using an analogous argumentation to the one the last example we obtain for $x < \min(x_0, x_1)$ an expression $f(\sqrt{r_1 x}) = \sqrt{r_2 k x}$. For sufficiently small x, precisely for x such that $\sqrt{r_1 x} < \min(x_0, x_1)$, we have $f(\sqrt{r_1 x}) = k\sqrt{r_1 x}$.

Thus for those x we obtain $\sqrt{r_2 k x} = k\sqrt{r_1 x}$, therefore $k = \frac{r_2}{r_1}$. However $k = \frac{x_2}{x_0} = \frac{r_2}{x_0}$. Thus $x_0 = r_1$ and finally we have $f(x) = kx = \frac{r_2}{r_1}x$ for $x < x_0 = r_1$.

The plots of functions m_1, m_2 and f with $r_1 = 1, r_2 = \frac{3}{2}$ are presented below.







6 Conclusion

In this paper we have introduced some problem related to fuzzy systems of control and then we have discussed some solutions of one functional equation which generalizes an equation connected with solutions of the distributivity equation of fuzzy implication functions over some classes of triangular conorms. At this moment it is quite difficult for us to show possible practical applications (in fuzzy logic) of such equations with other functions than minimum, but it is beginning of our work with such type of equations. Thus we believe that obtained results can be used in future not only in fuzzy logic but also in other theories like fuzzy mathematical morphology or aggregations functions, where solutions of functional equations play an important role.

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