Lecture

Title: Interaction Between Methods of Local Banach Space Theory and Dirichlet Series

Lecturer: prof. dr. hab. Andreas Defant

Time: 6 till 13 October (14 hours of lectures) and 17 till 24 November (16 hours of lectures)

Place: Wydział Matematyki i Informatyki UAM Poznań, ul. Umultowska 87

Biographical note: Prof. Adreas Defant is an Austrian mathematician working at University of Oldenburg (Germany). He is a specialist in functional analysis and its applications. He published over 80 mathematical papers. He is also an author of two monographs: *Classical summation in commutative and noncommutative Lp-spaces* (LNM 2021, Springer 2011) and *Tensor norms and operator ideals* (North Holland 1993; with K. Floret).

Andreas Defant studied mathematics and physics at the University of Kiel (1972-1979), where he got Diploma-degree in mathematics in 1979 and Doctor-degree in 1981. In 1987 he finished his habilitation in mathematics at University of Oldenburg. He works in several Universities: University of Kiel, University of Michigan (Ann Arbor), University of Zürich. From 1990 he has got a permanent position at the Mathematical Department of the University of Oldenburg. He lectures in many places, in particular in: Universita Degli Studi Di Lecce, Research Institut for Mathematics of the ETH Zürich, Universidade Federal do Rio de Janeiro, IMECC Unicamp, Campinas (Brasil), Technical University of Valencia, Technion Haifa Inst. of Math. of the Polish Academy of Sciences, University of Illinois, University of Leeds, Technical University of Valencia.

Description of the lecture

Local Banach space theory studies and classifies Banach spaces and the operators acting on them by so-called local properties. A property of a space is said to be local whenever it can be formulated by a quantitative statement holding for every choice of finitely many vectors or finite dimensional subspaces (for example, an inequality). Formulated differently, local Banach space theory tries to derive infinite dimensional statements on a given infinite dimensional Banach space and its operators by asymptotic considerations based on a systematic use of various finite dimensional concepts. Starting with Grothendieck's famous "Résumé" on the metric theory of tensor products, the last 60 years witnessed ground breaking research in this direction with an incredible impact on various fields in mathematics.

The aim of this series of lectures is twofold. One central aim is to present in a systematic way some basic features of local Banach space theory (as e.g. operator ideals, tensor products, Banach-Mazur distances, projection constants, type and cotype, ...).

But secondly, we intent to illustrate the power of all these concepts by applying them to ordinary Dirichlet series $\sum_n a_n n^{-s}$ and their close relatives, holomorphic functions on polydiscs. Contemporary research in this new topic owes much to the following fundamental observation of Bohr from 1913: By the transformation $p_j^{-s} = z_j$ (here p_j is the j-th prime number) and the fundamental theorem of arithmetics (for each integer there is a unique multi index α such that $n = p^\alpha$), an ordinary Dirichlet series $\sum_n a_n n^{-s}$ may be thought of as a function $\sum_\alpha a_{p^\alpha} z^\alpha$ of infinitely many complex variables $z_1, z_2, \ldots$.

Bohnenblust and Hille in 1931 used this revolutionary insight to solve a long-standing problem in the field: In 1913 Bohr had shown that the width of the strip on which a Dirichlet series converges uniformly but not absolutely is $\leq 1/2$, but Bohnenblust and Hille were able to prove that this upper estimate is even optimal. However, this research took place before the modern interplay between function theory and functional analysis, as well as the advent of the field of several complex variables, and the area was in many ways dormant until the late 1990's.

A new field is now emerging, intertwining the classical work of Bohr and Bohnenblust–Hille in novel ways with modern methods from functional analysis and in particular local Banach space theory. So far several fundamental questions have been settled, some long-standing problems have been solved, and a growing number of researchers with different backgrounds have become engaged in the area. As a result of this work, a number of challenging research problems have crystallized. Their solutions seem to require unconventional combinations of expertise from harmonic, functional, and complex analysis, as well as analytic number and probability theory.