Near-minimizers, Covering Theorems and Applications

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Lecture 1. **Calderón-Zygmund Decomposition and Real Interpolation**

1. Riesz rising sun Lemma and procedure Calderón-Zygmund procedure.
2. Calderón-Zygmund decomposition and its properties.
4. E-functional, near-optimal decomposition for E-functional. Connection with Calderón-Zygmund decomposition.

Lecture 2. **Some Topics in Real Interpolation**

1. K- and L- functionals, connection with E-functional. Simple properties.
2. Calculations for couple (L^1, L^∞).
3. Reiteration theorem for (θ, q) spaces and K-divisibility.
4. Equivalence and duality theorems.
5. Holmstedt formula for near-minimizers.

Lecture 3. **Applications to Harmonic Analysis**

1. A weak type inequality for linear operators and Hardy-Littlewood maximal operator.
2. Morrey-Campanato norms on BMO and Lipschitz spaces.
3. John-Nirenberg inequality.

Lecture 4. **Applications to Regularization Theory and Image Processing**

1. Tikhonov regularization and near-minimizers for the $L-$functional.
2. Rudin-Osher-Fatemi denoising model and exact minimizers for the $L-$ and $E-$functionals.

Lecture 5. **Classical Covering Theorems**

1. Whitney, Wiener and Besicovitch covering theorems.

Lecture 6. **Smooth Analoque of Calderón-Zygmund decomposition**

1. Construction and properties.

Lectures 7-8. **Stability of near-minimizers**

1. Couple (L^1, L^p), $1 \le p < \infty$.
2. Couple (L^1, L^∞).

Lecture 9. **Whitney-Besicovitch coverings and theorem on controlled coverings**

1. Whitney-Besicovitch coverings and partition of the unity.
2. Theorem on controlled coverings.

Lecture 10. **Construction of near-minimizers for spaces of smooth functions**

1. Couple $(L^p, \dot{W}^{k,p})$.
2. Couple $(L^p, \dot{W}^{k,q})$ and couples of Sobolev spaces.
3. Near-minimizers for Brudnyi and Triebel-Lizorkin spaces.

Course is based on the book:

S. Kislyakov and N. Kruglyak, *Extremal problems in interpolation theory, Whitney-Besicovitch coverings, and singular integrals,* Springer, 2013, 316 pp.