An Introductory Guide to References on Conservation Laws

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Disclaimer

These pages originate as a follow up to the course *Conservation Laws* delivered by the author at the University of Warsaw in the week 12-16.05.2014. Clearly, not all the theory of Conservation Laws is considered, several subjects are only superficially covered, while others are entirely omitted and the space devoted to the various research directions is not related to their relevance. Unfortunately, these choices reflect the author's limited knowledge, rather than his interests.

1 Text Books

The theory of Conservation Laws currently consists in the theory of 1D systems and in that of multiD scalar equations. A beautiful text comprising both is [146]. Here, several historical notes allow the reader to understand how the whole subject developed, keeping Euler equations as the guiding paradigm (non standard applications to, e.g., crowd dynamics or traffic flow are marginally considered). Moreover, a very complete bibliography helps finding all details not contained in this very clear exposition.

The monograph [58] is devoted to 1D systems. The uniqueness of solutions and their Lipschitz continuous dependence from the initial datum are presented in detail. The presence of exercises, the detailed explanations and the gentle introduction that covers also semilinear and quasilinear systems allow to use this text as a reference for a PhD course.

The two volumes [233] also deal with both multiD equations and 1D systems, although some results on the well posedness of the latter are not present. A variety of techniques is presented (Glimm scheme, compensated compactness, ...) and detailed references are given for those proofs that are not presented.

Peculiar to [209] is the thorough treatment of nonclassical shocks. Here, the proof of stability is obtained through a procedure different from that in [58].

The *Wave Front Tracking* approximation technique is the subject of the monograph [192], that also describes a 2D version. A text presenting numerical methods as well as theoretical results is [171], while [211] is entirely devoted to numerical methods. The monograph [219] is a very precise, mostly self contained and rigorous introduction to measure valued equations.

Prior to the breakthroughs that characterized the development of conservation laws after 1995, for several years the (first editions of the) books [230, 236] have played a very relevant role and were a main reference for this subject.

Other textbooks devoted to specific applications are [162], devoted to networks, and [228], devoted to crowd dynamics.

2 General Theory – 1D Systems

A good starting point on the general theory of conservation laws is the work [225] by Riemann. A century later came the solution to the Riemann problem by Lax [204], which opened the way the global in time existence of solutions in $\mathbf{BV}$ by Glimm [166]. An extension to the case of 2 equations with initial data in $\mathbf{L}^\infty$ was obtained in [167]. This result was slightly improved and its proof deeply simplified in [47].

The well posedness, i.e. the $\mathbf{L^1}$-Lipschitz continuous dependence from the initial datum, for 2×2 systems is proved in [61] by means of wave front tracking, in the $n \times n$ case in [64] using piecewise Lipschitz approximations. This latter proof was significantly simplified using wave front tracking and the Liu-Yang functional [71, 216, 217, 218], see also [25, 32, 48, 110, 196, 210]. Preliminar results that eventually lead to this breakthrough where [54, 56], apparently contradicting [238].

Remarkably, various results on the uniqueness of solutons and on their characterizations were obtained after continuous dependence was proved, often relying on the existence of the semigroup, see for instance [62, 65, 68, 70].

The *stability* of solutions can be understood in various senses. Their $\mathbf{L^1}$-Lipschitz continuous dependence from the flow is proved in [46]. A definition of structural stability is presented in [69].

Most of the results above are obtained under the assumption that each characteristic field be either linearly degenerate or genuinely nonlinear. This requirement is proved to be not necessary in the series of papers [23, 22, 24, 48, 196].

The above results are typically obtained through the definition of a sequence of approximate solutions. The exact solution is then proved to inherit properties of the approximate ones. Three widely used techniques to define approximate solutions are *Glimm Scheme*, *Wave Front Tracking* and *Vanishing Viscosity*.

Glimm Scheme: In its original form, presented in [166], it relies on a random choice and leads to probabilistic arguments. A deterministic version is presented in [214]. The uniqueness of the limit was proved only several years later in [57].

Wave Front Tracking: It originated in [142], was extended to $n \times n$ systems in [55, 226], is the central subject of the monograph [192] and of the lecture notes [59, 84]. A simplification is described in [34].

Vanishing Viscosity: The convergence of vanishing viscosity approximations has been a long standing question, finally closed in [45]. Remarkably, these approximations do not even require the system to be in conservative form. A rate of convergence for these approximations is obtained in [67].

Among the many qualitative properties of the solutions, decay estimates originated in [222] have been widely considered for their relevance in characterizing solutions, see [63, 72, 79, 165, 169, 210, 213].

Invariant regions are characterized in [189], see also [156, 157] for a numerical viewpoint.

A very effective tool in the qualitative analysis of solution is provided by generalized characteristics, originated in [143, 144], see also [147, 148].

The so called *Temple systems* [237] have a geometric structure that make them an optimal example, of interest also on their own. They have been and still are widely considered, see for instance [20, 106, 33, 41, 44, 66, 184].

A collection of open problems (at the time of the writing) is in [60].

Balance Laws Following [149], approximate solutions to the system of balance law $\partial_t u + \partial_x f(u) = g(t,x,u)$ are typically constructed through the *operator splitting* or *fractional step* algorithm. It consists of the alternate use of approximate solutions to $\partial_t u + \partial_x f(u) = 0$ with (approximate) solutions to the ordinary differential equation $\partial_t u = g(t,x,u)$. The development of this theory closely followed that of conservation laws, with existence of solutions being proved first in the 2×2 case in [140], then the $n \times n$ case and the Lipschitz continuous dependence from the data followed, see [8, 9, 10].

Further extensions are presented in [88, 108, 109, 113, 132, 133, 145, 170, 174]. General treatments of the nonlinear operator splitting algorithm in metric spaces are in [89, 111, 112].

Initial – Boundary Value Problems Assigning an initial datum and a boundary datum to a conservation law may well lead to an over-determined problem. The presence of a jump discontinuity between the boundary datum and the trace of the solution at the boundary puts under question the very sense under which the boundary data has to be understood. Depending on the specific application at hand, different definitions of solution can be preferable.

A general, entirely intrinsic definition of solution is proposed in [153, 172]. A well posedness theory developed along this line is found in [3, 4, 5, 106, 113, 132, 133, 152].

The vanishing viscosity approach yields a different theory, initiated in [18, 19] with artificial viscosity and then with real viscosity in [49], see also [77, 78].

3 General Theory – MultiD Scalar

Existence, uniqueness and $\mathbf{L^1}$-Lipschitz continuous dependence from the initial datum are proved in the classical paper [200] using the vanishing viscosity approach and the doubling of variables. A key role is played by the definition of solution. The same techniques are extended to the case with boundary in [37]. The stability of solution with respect to flow and source was proved more recently in [127, 128, 206, 207]. Differently from the usual habit in the case of 1D systems, these results all deal with the general case $\partial_t u + \mathrm{div} f(t,x,u) = g(t,x,u)$, i.e., with fluxes and sources depending on (t,x,u). Usually, this makes all statements rather intricate, since the regularity requirements on the dependence from t, x and u may well differ. Improvements of Kružkov result are, for instance, in [53].

4 Applications

4.1 Continuum Thermomechanics

The applications to Continuum Thermomechanics have been the paradigm that drove the evolution of the theory of Conservation Laws. Here, we only record the following research directions.

Phase Transitions: The idea underlining most of these works is that a phase boundary is nothing but an evolving free discontinuity between two different fluids. Since in Conservation Laws we know how to deal with discontinuities, we are also able to deal with phase transitions. Detonations, deflagrations and chemical reactions are treated similarly. It is difficult to draw a boundary separating the more mathematical works from the more physical ones: [1, 2, 6, 7, 27, 28, 75, 86, 87, 130, 136, 137, 138, 155, 183, 208, 234, 235].

Pipes, Junctions and Networks: Two pipes connected by a junction (or a kink) essentially originate an initial boundary value problem for a system of conservation laws where the two components related to the two pipes are coupled only through the junction. The physics of the junction lead to select the solution to the Riemann Problem at the junction and, hence, to the full analytic theory. This research direction initiated with [191], Irrigation canals can be treated similarly. See [35, 36, 51, 52, 93, 94, 95, 115, 119, 120, 121, 122, 125, 175, 215].

Granular Flows: Two models have been widely considered in the literature: the Hadeler–Kuttler one [182] and the Savage–Hutter one [231, 232]. More recently, new models were proposed, leading to new analytic problems: [11, 12, 13, 14, 15, 16, 17, 73, 116, 117, 176, 177, 178, 179, 180]. Related to these works, a description of the cutting of steel by means of a laser beam was recently proposed in [114].

4.2 Vehicular Traffic and Crowd Dynamics

These applications have been considered more recently. Besides the obvious conservation of the total amount of vehicles/individuals, they lack the presence of other well established basic principles. This is partly due to their rather young history and partly to the possible non existence of such principles.

Vehicular Traffic: A basic textbook, not specifically related to conservation laws, is [151]. The third part of the classic [181] is a gentle introduction to traffic modeling. The more recent [228] relies on conservation laws.

The most famous macroscopic model was presented in the famous works [212, 224]. A further classical reference is [221].

Also in vehicular traffic, 1995 has a particular role. Due to Daganzo's [150], the models proposed after that year have to cope with the deep criticism by Daganzo. A first reaction lead to the celebrated model [30, 241], with a lot of follow ups [29, 160, 168, 173]. Then various multiphase models also arose, as in [83] and [50, 75, 85, 99, 101, 124].

Traffic modeling naturally leads to several new analytic problems, such as optimal management [21, 102, 104, 105, 107]; coupling of different models [123, 163, 164, 202, 203]; nonlocal models [91, 118]; generalized models [31, 205]; multi-population models [42, 43, 240, 242]; multi-lane models [90, 193, 198, 199]; *ad hoc* numerical algorithms [75, 76]; see also [81, 82, 135, 187, 197, 201, 220, 229] and the reviews [39, 186].

A very quickly developing research direction is related to junctions and networks, originated in [190]. A good starting point is the monograph [162], whereas related papers are [26, 96, 100, 158, 159, 161].

Crowd Dynamics: This is an even newer research direction. Among the first papers devoted to the use of conservation laws in the modeling of crowd movements are [194, 195], see also [154], and the unusual [80]. The monograph [228] provides an introduction to 1D models, refer also to the special issue [129], to the review [39] and to the rather applied [239]. Nonclassical shocks were used in [131, 134, 227], which are in agreement with the experimental study [188], and in [103]; moreover, *ad hoc* numerical methods were developed [74]. Nonlocal equations seem particularly successful, see [92, 97, 98, 126, 141] also in the case of measure valued equations [223]. For other approaches, see [38, 40, 139, 185].

5 All the Rest

A sample of the subjects not covered above is the following: Numerical methods; Transport equations with low regularity; Non-uniqueness of solutions to multiD systems; Stability of shocks/rarefactions/contact discontinuities; Applications to structured population models; Kinetic formulations; Properties of viscous approximations; Systems with large data; Applications to general relativity; Nonclassical shocks; Compensated compactness methods; Regularity of solutions; Asymptotic decay estimates; Singular limits of parabolic approximations; Discontinuous fluxes; Control problems; ...

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