Dynamics of actions of groups and pseudogroups

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1 Introduction

1.1 Topological groups

Definition 1 *A topological space G which is also a group is called a semi-topological group if the mapping*

$$g_1 \colon G \times G \to G;\ \ (x, y) \mapsto x.y$$

is continuous in each variable separately.

Definition 2 *A topological space G which is also a group is called a topological group if the mapping*

$$g_1 \colon G \times G \to G;\ \ (x, y) \mapsto x.y$$

is continuous in both variables and the mapping

$$g_2 \colon G \to G;\ \ x \mapsto x^{-1}$$

is continuuos.

Proposition 1 *Any topological group is a semitopological group. There are semitopological groups which are not topological.*

Theorem 1 *(Montgomery) A complete and separable semitopological group is a topological group.*

Corollary 1 *A metric compact semitopological group is a topological group.*

Theorem 2 *A regular locally compact semitopological group satisfying the second axiom of countability is a topological group.*

Theorem 3 *A locally compact Hausdorff topological space that is also a group and in which the multiplication is continuous as a function of both variables is a topological group.*

1.2 Transformation groups

Let G be a group and X a topological space and let $a: G \times X \to X$ be a continuous mapping.

G acts continuously on a topological space X if for any elements g and h of G and any $x \in X$

$$a(g, a(h, x)) = a(gh, x)$$

Notation $a(h, x) = h(x)$
Then each mapping $a(h, .)$ is a homeomorphism of X, and the assignation $G \ni g \mapsto a(g, .)$ defines a representation of G into $Homeo(X)$.

We say that G is a transformation group of the topological space X.

Theorem 4 *An effective and transitive compact transformation group G acting on a locally connected finite dimensional space M is necessarily a Lie group.*

Theorem 5 *An effective and transitive compact transformation group G acting on a manifold M is necessarily a Lie group.*

1.3 Isometric actions

Let (X, d) a metric space.

A mapping $f: M \to M$ is called an isometry if for any

$$(*) \quad x, y \in X \quad d(f(x), f(y)) = d(x, y).$$

Group G acts by isometries if the condition (*) is satisfied for any $x, y \in X$ and $g \in G$. The natural representation of G takes values in $Isom(X, d)$

How big can $Isom(X, d)$ be?

Gao and Kechris, cf. [12], proved that every Polish group is isomorphic to the (full) isometry group of some separable complete metric space.

A Polish space is a separable completely metrizable topological space; that is, a space homeomorphic to a complete metric space that has a countable dense subset.

Let G be a topological group regarded as a topological space. Then G is defined to be a Polish group if it is also a Polish space.

When an action of a group can be made isometric?

Any action of a compact Lie group on a smooth manifold is isometric for some Riemannian metric.

1.4 Proper actions, cf. [22]

Consider $\{g \in G: gU \cap U \neq \emptyset\}$. We say that U is *thin* if that subset of G has compact closure.

A G-space X is a *Cartan G-space* if every point of X has a thin nbhd.

A subset S of a G-space X is a *small subset* of X if each point of X has a nbhd U_x which is thin relative to S, i.e. $\{g \in G: gU_x \cap S \neq \emptyset\}$ has compact closure.

A G-space is *proper* if each point of X has a small nbhd.

Remark *A proper G-space is a Cartan G-space.* (Converse is false)

If X is a Cartan G-space then each orbit is closed in X and each isotropy group is compact.

Proposition 2 *If X is a proper G-space then X/G is completely regular (Tychonoff).*

Let H be a closed subgroup of G. A subset S of X is called an H-kernel if there exists an equivariant map $f: GS \to G/H$ such that $f^{-1}(H) = S$. If in addition GS is open in X S is called an H*-slice* in X. If $x \in X$ then by a slice at x we understand a G_x- slice in X.

Theorem 6 *Let G be a Lie group, X a G-space and $x \in X$. The following two conditions are equivalent:*

(1) G_x is compact and there is a slice at x;

(2) there is a nbhd V of x in X such that $\{g \in G: gV \cap V \neq \emptyset\}$ has compact closure in G.

Theorem 7 *Let G be a Lie group, X a G-space and $x \in X$. The following two conditions are equivalent:*

(1) G_x is compact and there is a slice at x;

(2) X is a Cartan G-space.

1.5 Equicontinuous actions

Isometric actions are closely related to the chosen metric. If we change the metric, even for an equivalent one, the action can cease to be isometric. It satisfies the following condition:

$$for\ any\ \epsilon > 0\ \ there\ exists\ \delta > 0\ such\ that\ for\ any\ x, y \in X\ \ and\ g \in G$$

$$d(x, y) < \delta \ \ \Rightarrow d(gx, gy) < \epsilon$$

The action satisfy this condition for any equivalent metric.

Any equicontinuous action satisfies also a bit weaker (?) condition:

for any two distinct points x, y of X $inf_{g\in G}\{d(gx, gy)\} > 0$

The action of G on X satisfying the above condition is called *distal* . The property "distal" is an invariant of equivalent metrics.

An equivalent condition

The action is distal unless for a pair of distinct points x, y of X there exists a sequence of g_n in G and a point z of X such that $g_n x \to z$ and $g_n y \to z$.

Remark Any equicontinuous action is distal.

Do non-equicontinuous distal actions exist?

Example (Auslander) cf. [5], p. 55.

Let N be a connected, simply-connected, nilpotent Lie group and D be a uniform, discrete subgroup such that $M = N/D$ is a nilmanifold. Let ϕ be a one-parameter subgroup of N and let ϕ^* be the induced flow on M. Then ϕ^* is distal.

2 Distal actions

2.1 Which groups act distally? cf. [1, 2, 3]

Theorem 8 *An almost connected locally compact second countable topological group admits an effective distal action iff it is polynomially growing.*

Theorem 9 *Every polynomially growing connected Lie group G can be embedded as a closed normal subgroup into a connected Lie group, which has a closed connected normal nilpotent subgroup with compact factor group.*

Corollary 2 *Every connected nilpotent Lie group has an effective distal action.*

Theorem 10 *Let H be a group acting by automorphisms on the connected Lie group G, Let ρ be the corresponding linear action of H on the Lie algebra g of G. Then the following conditions are equivalent:*

a) H acts distally on G.

b) The linear action ρ of H on g is distal.

c) The closures of H-orbits in G are minimal H-spaces.

Proposition 3 *Let the Lie group H with finite component group act continuously and linearly on a finite dimensional real vector space, let $\rho\colon H \to gl(V)$ be the corresponding homomorphism of Lie algebras. Then the following conditions are equivalent*

a) The action of H on V is distal.

b) The eigenvalues of all elements of $\rho(X)$, $X \in h$, are purely imaginary.

Theorem 11 *Let G be a group acting affinely on an affine space M. Then the following statements 1) and 2) are equivalent and imply 3). Moreover, if G acts linearly with respect to some point $x \in M$, then 3) is equivalent to 1) and 2).*

1) G acts distally.

2) The eigenvalues of the linear part of the affine maps corresponding to the elements of G are of absolute value one.

3) The closures of the G-orbits are minimal G-spaces.

Theorem 12 *Let V be a finite dimensional real vector space. Suppose G is a distal subgroup of $GL(V)$ and V is an irreducible G-module. Then G has compact closure.*

Theorem 13 *Suppose G is a subgroup of $GL(V)$. Then G is distal iff there is a flag $V = V_0 \supset V_1 \supset \ \supset V_n = (0)$ of G-submodules of V such that G acts isometrically on each quotient space V_i/V_{i+1}.*

2.2 Distal flows

We assume that (X, d) is a compact metric space and T a locally compact topological group.

By a (T-)flow (X, T) we understand a continuous action

$$T \times X \to X$$

such that for any $s, t \in T$ and any $x \in X$ $s(tx) = (st)x$

Definition 3 *Let (X,T) and (Y,T) be two flows and $p\colon X \to Y$ a surjective map (continuous). We say that (Y,T) is a subflow of (X,T) ((X,T) is an extention of (Y,T) relative to p) if for any $t \in T, x \in X$ $p(tx) = tp(x)$.*

Notation $(Y, T) = p(X, T)$

Two flows (X, T) and (Y, T) are said to be isomorphic if (Y, T) is a subflow of (X, T) relative to p and the map p is a homeomorphism. Otherwise we call (Y, T) a proper subflow of (X, T), and (X, T) a non-trivial extension of (Y, T).

Trivial flow (X_0, T) if $X_0 = \{x_0\}$ and T acts by identities.

Let (X, T) be a flow, consider two subflows $(Y_1, T) = p_1(X, T)$ and $(Y_2, T) = p_2(X, T)$. Assume that there exists a map $p'\colon Y_1 \to Y_2$ such that $p_2 = p'p_1$.

If such a mapping exists, then it is unique. p' determines (Y_2, T) as a subflow of (Y_1, T).

We have a partial order $p_2(X, T) < p_1(X, T)$ if $p_2 = p'p_1$. for some p'.

Definition 4 *A flow (X, T) is called minimal if no closed proper subset of X is invariant under the action of T.*

2.3 Isometric extensions, cf. [11]

Definition 5 *Let X and Y be two compact metric spaces, a map* $p: X \to Y$, *and M a homogeneous compact metric space. We say that X is an M-bundle over Y if there exists a real valued function* $\rho(x, y)$ *defined whenever* $p(x) = p(y)$ *and such that*

(a) ρ *is continuous (on the subset of* $X \times X$ *of pairs (x,y) such that p(x)=p(y));*

(b) for each $y \in Y$ ρ *defines a metric on the fibre* $X_y = p^{-1}(y)$ *with which* X_y *is isometric to M.*

Definition 6 *Let X be an M-bundle over Y with* $p: X \to Y$, *let (X,T) be a flow and (Y,T) = p(X,T) a subflow. We say that (X,T) is an isometric extension of (Y,T) if the function* ρ *satisfies the condition*

$$\rho(tx, ty) = \rho(x, y)$$

for any x,y in the same fibre and any $t \in T$.

Proposition 4 *An isometric extension of a distal flow is distal.*

Definition 7 *Let (X,T) be a flow and let* $\mathcal{X} = \{(X_\alpha, T)\}$ *be a collection of subflows of (X,T),* $(X_\alpha, T) = p_\alpha(X, T)$. *We sat that (X,T) is a limit of the flows in* $\mathcal{X}$ *if for every pair of distinct point x,y of X there exists a subflow* (X_α, T) *such that* $p_\alpha(x) \neq p_\alpha(y)$.

Proposition 5 *A limit of distal flows is distal.*

Definition 8 *Let (X,T) be a flow and (Y,T) a subflow. Suppose there is an ordinal* η *such that for each ordinal* $\xi \leq \eta$ *there is associated a subflow* (X_ξ, T) *of (X,T) such that the following are satisfied:*
(a) $(X_0, T) = (Y, T)$ *and* $(X_\eta, T) = (X, T)$ *where 0 denotes the least ordinal.*
(b) If $\xi < \xi', (X_\xi, T) < (X_{\xi'}, T)$.
(c) For each $\xi < \eta, (X_{\xi+1}, T)$ *is an isometric extension of* (X_ξ, T).
(d) If ξ *is a limit ordinal* $\leq \eta$, *then* (X_ξ, T) *is the limit of* $\{(X_{\xi'}, T), \xi' < \xi\}$.
We then say that (X,T) is a quasi-isometric extension of (Y,T).

Definition 9 *A quasi-isometric extension of a trivial flow is called a quasi-isometric flow (q.i. flow)*

Theorem 14 *Every quasi-isometric flow is distal.*

2.4 The Ellis group, cf. [11]

Let (X,d) be a compact metric space. By X^X we understand the space of all transformations of X into X, continuous or not. X^X is a compact Hausdorff topological space with respect to the topology defined as the weakest topology rendering all evaluation mappings continuous ($\hat{x}(f) = f(x)$). A subbase is this topology is given by the sets of the form

$$U_\epsilon(x, y) = \{f \in X^X \colon d(x, f(y)) < \epsilon\}$$

It is the standard product topology.

For a fixed f_0 the map $f \mapsto ff_0$ is continuous.

The inverse image of $U_\epsilon(x, y)$ is equal to $U_\epsilon(x, f_0(y))$

The map $f \mapsto f_0 f$ is not continuous in general unless f_0 is continuous

If (X, T) is a flow, T can be mapped into X^X. If the action is effective, then this mapping is injective. Its closure we denote by Γ. Γ is compact and a semigroup.

Theorem 15 *If (X,T) is distal, then Γ is a group.*

Definition 10 *The group Γ is called the Ellis group of the distal flow (X,T). The induced topology is called the E-topology.*

Theorem 16 *If (X,T) is distal then X is the disjoint union of the minimal sets in X of the flow.*

Corollary 3 *If (X,T) is minimal distal, then its Ellis group Γ is transitive on X.*

Example (The Ellis group of a distal but not equicontinuos flow)

Take X- the 2-torus, T = Z - the group of integers. Let the flow be given by

$$\tau(x_1, x_2) = (e^{i\alpha} x_1, \psi(x_1) x_2)$$

where α is an irrational multiple of π.

The Ellis group consists of all transformations

$$\gamma(x_1, x_2) = (e^{i\beta}x_1, \phi(x_1)x_2)$$

where β is real and ϕ is any function of x_1 with $|\phi(x_1)| = 1$ satisfying the condition

$$\frac{(e^{i\alpha}x)}{\psi(x)} = \frac{\phi(e^{i\beta}x)}{\phi(x)}$$

It is a serious restriction if ψ were a continuous function. Otherwise ψ may be multiplied by any function on the circle group of unit modulus which is constant on cosets of the subgroup $\{e^{in\alpha}\}$

First, we enumerate some basic properties of the Ellis semigroup, cf. [10, **?**].

i) For any $x \in M$, the evaluation mapping $\hat{x}\colon \Gamma \to M$ is continuous;

ii) For any $\gamma \in \Gamma$, the right translation $R^\gamma\colon \Gamma \to \Gamma, \Gamma \ni f \mapsto f\gamma$, is continuous;

iii) For any $\gamma \in \Gamma$, a continuous transformation of M, the left translation $L_\gamma\colon \Gamma \to \Gamma, \Gamma \ni f \mapsto \gamma f$, is continuous. So in particular L_γ is continuous for any $\gamma \in T$;

iv) If the action σ is distal, then Γ is a group, but not, in general, a topological group.

v) For a distal action, the Ellis group is a topological group iff the action is equicontinuous.

vi) If the action $\sigma\colon T \times M \longrightarrow M$ extends to a continuous and effective action $\overline{\sigma}\colon G \times M \longrightarrow M$ of some compact group G ($T \subset G$), then the Ellis group of the action σ can be identified with $\overline{T} = H \subset G$.

Theorem 17 *Every minimal distal flow is quasi-isometric.*

2.5 Further properties

Theorem 18 *A subflow of a distal flow is distal, and the Ellis group of a subflow is the homomorphic image of the Ellis group of the flow.*

Theorem 19 *If X is simply connected, then X does not admit any minimal distal flow for any locally compact abelian group T.*

Theorem 20 *If (X, T) is a distal flow where T is an arbitrary locally compact group, then there exists a probability measure on X invariant under T.*

3 Smooth group actions

3.1 Abelian group actions

Imagine that a Lie group G acts locally freely via σ on a compact manifold M. Assume that this action is distal. This action defines a regular foliation $\mathcal{F}_G$ of M. Let σ' be another action of the group G with the same orbit. Is that action distal?

The answer is very complicated. In some cases **YES** and in some **NO**

Example Flows on 2-tori.

Consider the standard linear flow on the 2-torus. Its leaves, orbits, are either closed or dense. In the first case the flow is given by parallel lines in the plane with rational slope, in the second with irrational one. However, in some cases all R-actions with these orbits are conjugated, and in some this is not true.

Can we distinguish these two cases by some means? For example by some kind of cohomological invariants?

YES

Definition 11 *The irrational number c is called Liouville if for any integer n there exist integers p and q, relatively prime such that $|c - \frac{p}{q}| < \frac{1}{q^n}$. Otherwise it is called diophantine.*

If the slope is Liouville then any R-actions having the same orbits are conjugated, thus distal.
If the slope is diophantine then there are actions which are not conjugated to the standard one, some of them are not-distal.

A. Haefliger, [15], showed that in the first case the foliated cohomology in dimension 1 is of dimension one, and in the other case infinite dimensional.

It is not a coincidence!!

3.2 Foliated cohomology

Let $(M, \mathcal{F})$ be a foliated manifold, let F be the tangent bundle to the leaves of F. Let $A^*(M/\mathcal{F})$ be the complex of smooth forms along the leaves, i.e. global smooth sections of the exterior power of the cotangent bundle F^* along the leaves. The d_F be the differential operator which differentiates only along the leaves. The cohomology of such a complex $A^*(M/\mathcal{F}), d_F$ is called the foliated cohomology of the foliated manifold $(M, \mathcal{F})$.

As many examples show this cohomology can be infinite dimensional even for relatively simple foliations. Moreover, the Künneth formula is not always applicable, cf. [6].

3.3 Rigidity

Let $\sigma\colon M \times G \to M$ be right C^r-action

We say that σ is

A) C^r*-locally rigid* if there exists an open nbhd $\mathcal{U}$ of σ such that for any $\kappa \in \mathcal{U}$ there exist a diffeomorphism F of M and an automorphism Φ of G such that

$$F(\sigma(x, g)) = \kappa(F(x), \Phi(g))$$

B) C^r*-locally orbit rigid* if there exists an open nbhd $\mathcal{U}$ of σ such that for any $\kappa \in \mathcal{U}$ there exists a diffeomorphism F of M and an automorphism Φ of G such that

$$F\colon (M, \mathcal{F}_\sigma) \to (M, \mathcal{F}_\kappa)$$

is a foliated map.

C) is *parameter rigid* (PR) if for any $\kappa\colon M \times G \to M$ such that $\mathcal{F}_\sigma = \mathcal{F}_\kappa$ there exist an automorphism Φ of G and a leaf-preserving diffeomorphism F of M isotopic to the identity via such diffeomorphisms satisfying

$$F(\sigma(x, g)) = \kappa(F(x), \Phi(g))$$

Assume that the manifold M is compact and the action σ is (PR). If σ is a distal then any of its re-parametrizations is distal as well.

Let G be a contractible Lie group and M be a closed, orientable, connected m-manifold and A be a right LF smooth action of G on M. The action A gives a homomorphism of G into the group of automorphisms of the ring $C^1(M)$ of the smooth functions on M given by

$$gf(x) = f(A(x, g))$$

for $g \in G$ and $f \in C^1(M)$,

defining a G-module structure $C^1_A(M)$ on $C^1(M)$. The cohomology of the action A is by definition the cohomology $H(G, C^1_A(M))$ of the G-module $C^1_A(M)$. The first cohomology group plays a fundamental role in the study of LF actions, and we recall the definition of this group.

A 1-cocycle over the action $A(x,g) = xg$ is a smooth mapping $a : MG \to R$ which satisfies the equations

$$a(x, gh) = a(x, g) + a(xg, h)$$

for any $x \in M$ and $g, h \in G$.

We call a a 1-coboundary if there exists a smooth mapping $b : M \to R$ such that

$$a(x, g) = b(x) + b(xg)$$

for $x \in M$ and $g \in G$.

The set $Z^1(G, C^1_A(M))$ of all 1-cocycles over A is a vector space and the set $B^1(G, C^1_A(M))$ of all 1-coboundaries is a vector subspace of the vector space of all 1-cocycles. The first cohomology group $H^1(G, C^1_A(M))$ is the corresponding quotient space.

Definition 12 *A LF action $A : MG \to M$ is CR1 if every 1-cocycle $a : MG \to R$ over A is cohomologous to a homomorphism $\Phi : G \to R$, i.e. there exists a smooth function $b : M \to R$ such that*

$$a(x, g) = b(x) + \Phi(g) + b(xg).$$

We note that the action A is CR1 if and only if $H^1(G, C^1_A(M))$ is isomorphic to the first cohomology $H^1(g^*)$ of the Lie algebra g of G. In fact, $Hom(G, R) \equiv Hom(g, R)$ since G is contractible and $Hom(G, R) \equiv Hom(G/[G, G], R) \equiv H^1(g^*)$.

Given two smooth free actions σ and κ of the same Lie group G defining the same foliation there exists a smooth map

$$\alpha : M \times G \to G$$

such that

$$\kappa(x, \alpha(x, g)) = \sigma(x, g)$$

The map α is a cocycle for the action σ, i.e.

$$\alpha(x, gh) = \alpha(\sigma(g, x), h), \alpha(x, g)$$

Given an orbit-preserving diffeomorphism F, isotopic to the identity, there exists a smooth map $b: M \to G$ defined as

$$F(x) = \kappa(x, b(x)^{-1})$$

Putting things together we get

$$F(xg) = \kappa(F(x), \Phi(g))$$

and hence

$$\alpha(x, g) = b(x)^{-1}\Phi(x)b(xg)$$

D) is *cocycle rigid* if given any smooth map $\alpha: G \times M \to G$ satisfying

$$\alpha(x, gh) = \alpha(g, x)\alpha(\sigma(x, g), h)$$

there exists a homomorphism Φ of G and a smooth map $b: G \to G$ such that

$$\alpha(x, g) = b^{-1}(x)\Phi(g)b(\sigma(x, g)$$

Remarks (D) implies (C) (Proposition 2.3, [18]) and for $G = R^k$ these two conditions are equivalent.

E) is *cohomology rigid in dimension 1* (CR1) if the first cohomology of σ if isomorphic to $H^1(g^*)$.

Proposition 6 *Let $\sigma: M \times G \to M$ is a right locally free smooth action. If G is contractible,*
$H^1(G, C^\infty_\sigma(M))$ is isomorphic to $H^1(M/\mathcal{F}_\sigma)$.

The condition (CR1) for locally free actions of Heisenberg is weaker than the cocycle rigidity.

Theorem 21 *For any locally free action $\sigma: G \times M \to M$ be a smooth action of the Heisenberg group $G = H_n$ on a compact smooth manifold M. If the action is (CR1), then it is parameter rigid.*

Theorem 22 *For any locally free action* $\kappa\colon R^k \times M \to M \quad dim H^1(M/\mathcal{F}_\kappa) \geq k$.

Theorem 23 *For any locally free action* $\kappa\colon R^k \times M \to M \quad dim(M/\mathcal{F}_\kappa) = k$. *iff the action* κ *is parameter rigid.*

4 Dynamics of pseudogroups

In the foliation theory or that of foliated spaces or laminations the dynamics of the structure can be read from the dynamics of its holonomy pseudogroup. The best reference are

a) P. Walczak, Dynamics of Foliations, Groups and Pseudogroups, Monografie Matematyczne, Vol. 64, Birkhäuser, Basel 2004.

b) C. C. Moore, Cl. L. Schochet, Global Analysis on Foliated Spaces, Cambridge University Press, 2005.

Let $\mathcal{F}$ be a foliation on a Riemannian n-manifold (M, g). Then $\mathcal{F}$ is defined by a cocycle $\mathcal{U} = \{U_i, f_i, g_{ij}\}_{i\in I}$ modelled on a q-manifold N_0, where

- $\{U_i\}_{i\in I}$ is an open covering of M,
- $f_i\colon U_i \to N_0$ are submersions with connected fibers,
- $g_{ij}\colon N_0 \to N_0$ are local diffeomorphisms of N_0 such that $f_i = g_{ij} f_j$ on $U_i \cap U_j$.

The connected components of the trace of any leaf of $\mathcal{F}$ on U_i consist of fibers of f_i. The open subsets $N_i = f_i(U_i) \subset N_0$ form a q-manifold $N = \coprod N_i$, which can be considered to be a transverse manifold of the foliation $\mathcal{F}$. The pseudogroup $\mathcal{H}_N$ of local diffeomorphisms of N generated by g_{ij} is called the *holonomy pseudogroup* of the foliated manifold $(M, \mathcal{F})$ defined by the cocycle $\mathcal{U}$.

Two cocycles defining the same foliations generate equivalent holonomy pseudogroups.

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