Topics in the Calculus of Variations
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— University of Warsaw, December 2012 —

This fifteen-hour compact course will provide a rapid introduction to a variety of modern techniques and results in the fields of Calculus of Variations and Analysis, with the eye on applications centered around the Mathematical Theory of Elasticity.

The course will be self-contained. A tentative outlay of the lectures is as follows:

<u>Lectures 1 - 4:</u>

- The notion of Γ-convergence and its fundamental properties.
- Examples of Γ-convergence in the linearized elasticity.
- The fundamental role of Korn's inequality.
- Nonlinear elasticity of shells with arbitrary geometry: derivation of the Von Karman energy as the Γ-limit of the nonlinear energies.
- The fundamental role of the geometric rigidity inequality.

<u>Lectures 5 - 8:</u>

- A proof of Korn's inequality.
- A proof of the Friesecke-James-Muller geometric rigidity inequality.

<u>Lectures 9 - 10:</u>

- The hierarchy of the limiting theories for elastic plates and shells. Energy scaling.
- The matching properties.
- A proof of the matching property for convex shells.

<u>Lectures 10 - 15:</u>

- The Aviles-Giga functional and a compactness result in the gradient theory of phase transitions.
- The Div-Curl lemma (a self-contained proof due to Polisevskii).
- The Young measures and their basic properties.
- The proof of the Desimone-Muller-Kohn-Otto compactness result.

Prerequisites: Knowledge of Lebesgue measure and integration, Lebesgue spaces L^p and basic knowledge of Sobolev spaces $W^{1,p}$, as well as the standard linear algebra material and the basic differential geometry in $\mathbb{R}^3$ will be assumed.