Computational Topology and Dynamics

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April 21, 2010

The focus of this course is the development of Conley theory as a mathematically rigorous, robust, and computationally effective framework for applied dynamics. A tentative outline is as follows (the goal is to cover parts 1-3 in the first two week section):

1. **Introduction.** This includes examples of nonlinear dynamics arising from finite and infinite dimensional maps, ordinary and partial differential equations, and experimental time series data to motivate the need to a topological approach to applied dynamics. We will also introduce basic concepts from dynamical systems.

2. **Decomposing Dynamics.** This includes Attractor-Repeller pair decompositions, Lyapunov functions, Morse Decompositions, Isolated Invariant sets and the associated lattice structures.

3. **Combinatorial Dynamics.** Topics covered will include multivalued maps, combinatorialization of dynamical systems

4. **Convergence.** The combinatorial dynamics and decomposition results will be viewed as an approximation scheme that provides an algorithmic approach to Conley's fundamental decomposition theorem.

5. **Contination.** We will discuss the sheaf structures associated with the above mentioned topics as a means of quantifying the robustness properties of Conley theory. We will also relate this to classical bifurcation theory.

6. **Conley Index.** This includes a discussion of index pairs, index maps, shift equivalence connected simple systems used in the definition of the index. We will also discuss how the index can be used to understand existence and structures of dynamical systems.

7. **Computing Homology.** Time permitting we will discuss algorithms for computing homology.

Through out the course we will present examples to motivate and explain the topics and algorithms to indicate the computability of this approach.