LECTURE #1
Roles of LPT List Scheduling on Parallel Machines

In this lecture, we discuss a popular List Scheduling technique, in particular its LPT variant applied to scheduling on parallel machines, identical or uniform, where the latter may have different speeds. The objective function is the makespan, i.e., the maximum completion time. A good review of the corresponding models is contained in [3].

List Scheduling is an approach to finding an approximate non-preemptive schedule on parallel machines, initially due to Graham [4]. We present and analyze the approach for identical machines and an arbitrary jobs list. Then an algorithm based on the LPT (Longest Processing Time) list with an improved performance is discussed, based on [2, 5]. We stress the role of LPT schedules for determining the power of preemption, defined as the ratio of the makespan for the optimal non-preemptive schedule to the makespan of the optimal preemptive schedule. For identical parallel machines, we present the power of preemption results obtained in [1, 6, 7], while for uniform machines the results obtained in [8, 9, 10].

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LECTURE #2
Linking Scheduling to Submodular Optimization

Submodular Optimization is one of the most advanced topics of combinatorial optimization [1]. One type of problems studied in submodular optimization is related to optimizing submodular functions, while another type includes problems with a feasible region described by submodular constraints, such as, e.g., a polymatroid.

In this lecture, we briefly review the results for problems of the first type. We then demonstrate how problems of scheduling with controllable processing times can be formulated in terms of optimization problems over a submodular polyhedron intersected with a box, and discuss approaches to solving these scheduling problems based on submodular optimization reasoning [2, 3].

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LECTURE #3
Scheduling and Boolean Quadratic Programming

In this lecture, we discuss links between a range of single machine scheduling problems with min-sum objectives and problems of Boolean programming with a quadratic objective function, related to the Half-Product that is is a specific quadratic function introduced in [1].

We review the techniques that lead to a fully polynomial-time approximation scheme (FPTAS) for the half-product [2] and its versions, including the positive half-product and the symmetric quadratic knapsack problem [3, 4, 6]. Based on [5], we also demonstrate how scheduling problems can be reformulated in terms of problems of minimizing a function similar to the half-product, with and without the knapsack constraint.

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LECTURE #4
Applications of Birkhoff–von Neumann Theorem in Scheduling Theory

In this lecture, we will consider applications of Birkhoff–von Neumann theorem on decomposition of a doubly stochastic matrix, which has multiple applications in various areas of mathematics and statistics [3]. The focus of the lecture is on the theorem's applications to some scheduling problems.

We begin with recalling the theorem and outlying its constructive proof. Next, following an approach developed in [1, 6], we show how to use the theorem for finding optimal preemptive schedules for the open shop scheduling problem and, following [1, 2, 5, 6], for scheduling problems with unrelated parallel machines. We complete the lecture by derivation of the "wrap-around" McNaughton's algorithm [4], not from the first principles, but based on Birkhoff–von Neumann theorem and the results from [1, 4].

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LECTURE #5
Geometric Methods in Scheduling

In this lecture, we discuss a problem of compact vector summation and its applications to flow shop and open shop scheduling. The content of the lecture gives a rear example when the results of classical mathematical analysis form a basis for scheduling algorithms.

We start with the classical Steinitz Lemma that asserts that it is possible to renumber the vectors of a given family in such a way that at any time their partial sum belongs to a ball of a certain radius, provided that each vector belongs to a unit ball. The history of the lemma from Riemann via Steinitz to modern times is described in [1, 3].

Based on [2, 6], we present a constructive proof of Steinitz Lemma which allows finding the required vector numbering in polynomial time. Further, we show how Steinitz Lemma can be applied to deliver approximation algorithms for the flow shop scheduling problem to minimize the makespan, either with a bounded absolute error [4, 6] or a constant relative error [5]. Finally, we use Steinitz Lemma to demonstrate that the open shop scheduling problem to minimize the makespan can be solved in polynomial time provided that the maximum machine load is considerably large.

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